# PLATE 2ND.

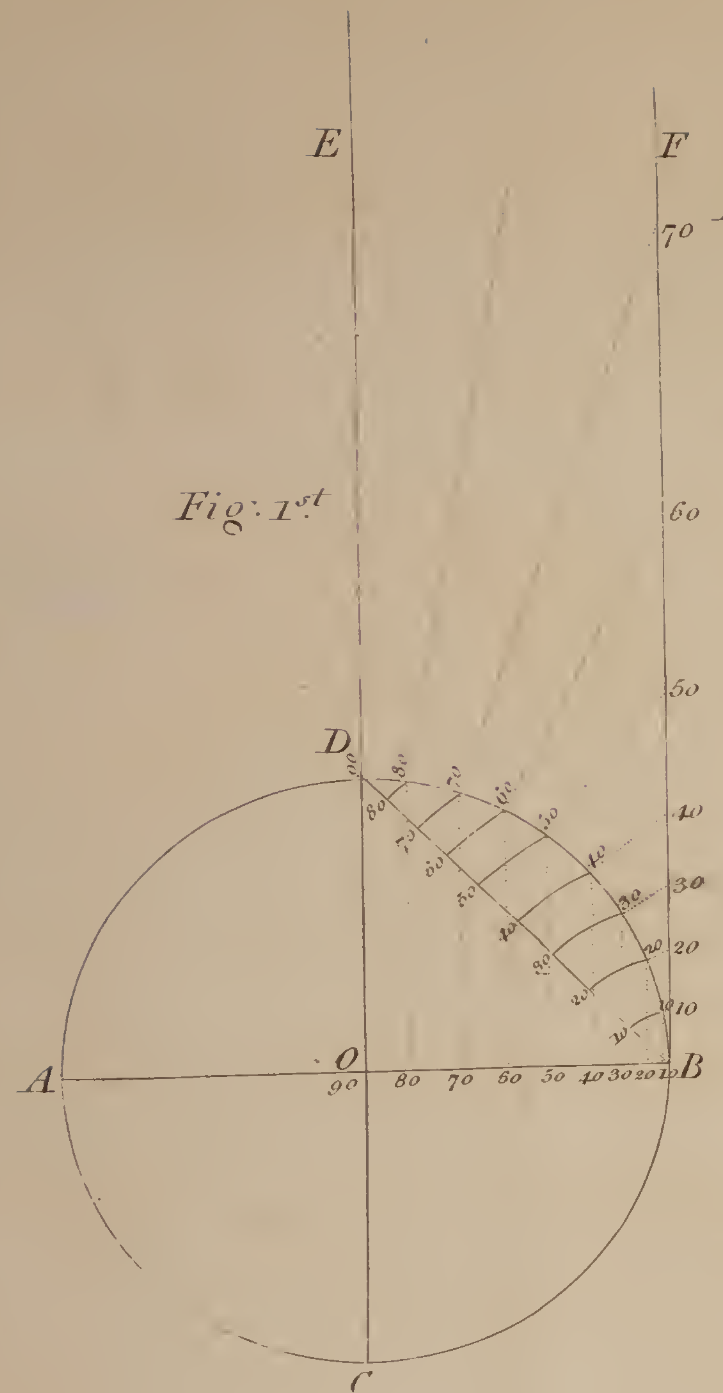
Fig. 1st

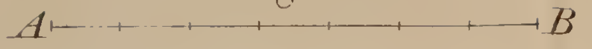
Fig. 2nd

Fig. 3rd

Fig. 4th

Fig. 5th

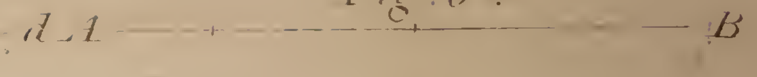
Fig. 6th

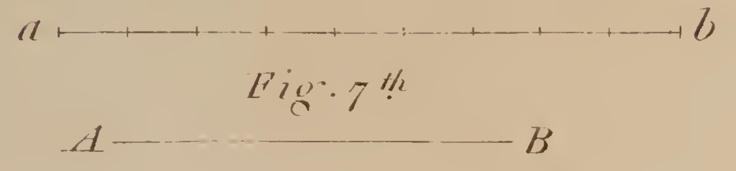
Fig. 7th

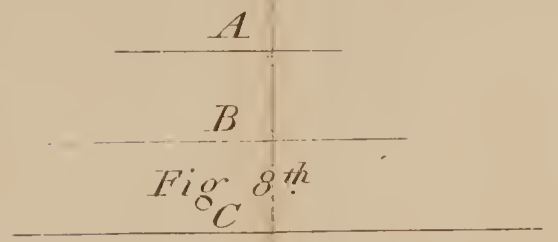
Fig. 8th

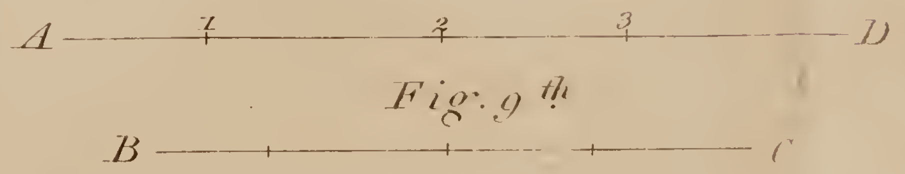
Fig. 9th

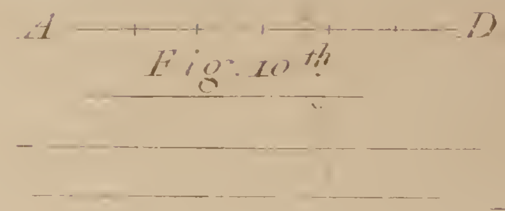
Fig. 10th

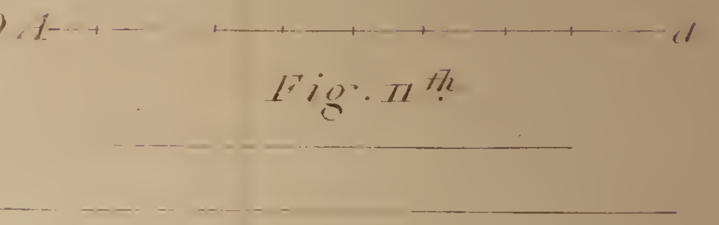
Fig. 11th

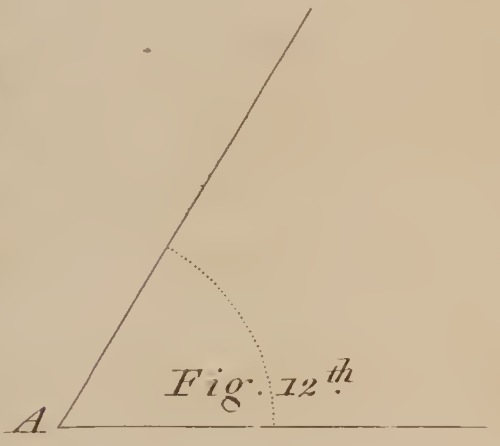
Fig. 12th

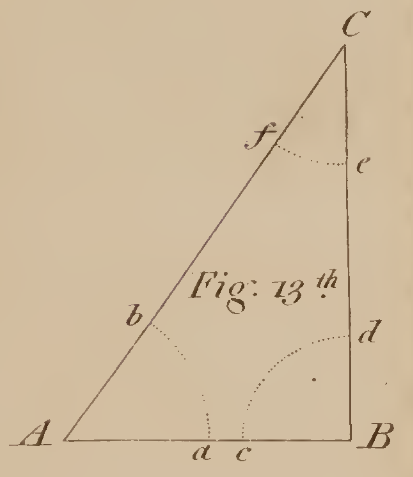
Fig. 13th

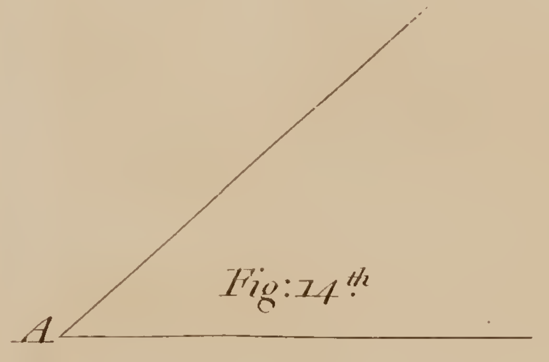
Fig. 14th

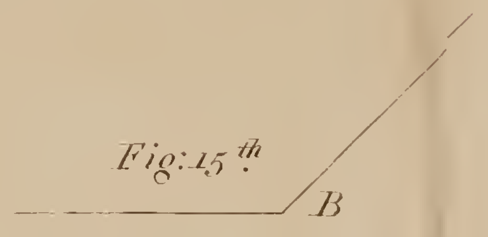
Fig. 15th

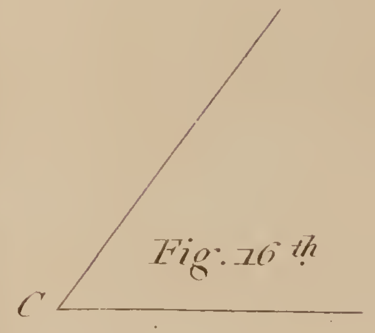
Fig. 16th

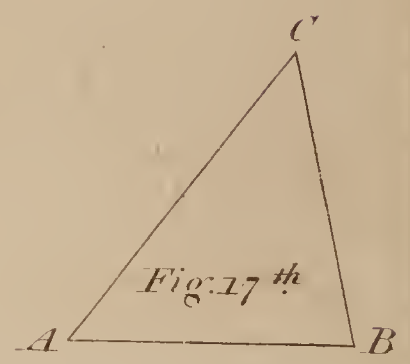
Fig. 17th

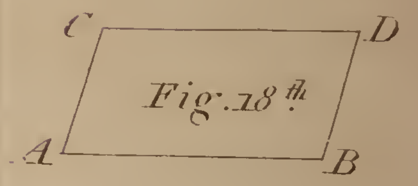
Fig. 18th

London, Published by J. Souter, 73, St. Pauls Ch. Yard, March 25th 1823.

3

A

# BRIEF TREATISE

ON THE

USE AND CONSTRUCTION

OF

# A CASE

OF

# INSTRUMENTS,

FOR THE USE OF SCHOOLS.

---

SECOND EDITION.

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BY

**G. PHILLIPS, B. A.**
Queen's College, Cambridge.

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ON THE

# USE AND CONSTRUCTION

OF

# A CASE OF INSTRUMENTS,

&c. &c.

## DESCRIPTION OF THE COMPASSES.

THE larger pair of compasses consists of two legs, turning upon a pivot at one end; one of the legs is screwed into a socket, and is frequently taken out for the purpose of fixing in a brass pen or pencil, to describe circles, and curvilineal figures, the purpose for which these compasses are principally used.

The smaller pair, sometimes called dividers, is a pair that also consists of two legs, and turns upon a pivot at one end, both legs are fixed; the chief use of this pair is to lay off angles and distances.

The beam-compasses is a small pair, which has but one leg, and a brass pen fixed to the other part, turning upon a pivot, at the end of which is a handle fixed about one-third of the length of the compasses; the design of this pair is to describe small circles, &c., which can be done with greater accuracy than with the larger compasses.

*Note.*—The triangular compasses are used in copying plans, maps, &c.; by means of which a person can take off the three points of any triangle that may be required.

The brass pen is an instrument which has two parts fixed in the handle, and are screwed together in the middle. The handle also consists of two parts, one screwed into another; the end of one of these parts is a point used for pricking on maps, charts, &c. The use of this instrument is to draw rectilineal lines, &c.

There is another pen, made in the same manner as the former, but has no handle, one end being made so that it can be fixed into the compasses for the purpose already mentioned.

The brass pencil is an instrument made so that one end is adapted for a pencil to be fixed in, and the other so that it can be placed in the compasses, used in the same manner as the brass pen.

The dotter is an instrument about the size of the brass pen; at one end is a small wheel, which turns on a little pin put through its centre, and is thus connected to the other part.

## DESCRIPTION AND USE OF THE SCALE.

The instrument we are about to explain is of great utility. I shall begin first to describe the scale, called the scale of equal parts.

There are generally on one side or face of this instrument six scales, of equal parts, of different lengths for convenience, each containing as many divisions as can be drawn, reckoning from the left hand towards the right, and the first division is subdivided into ten equal parts. The above scales are drawn parallel to each other, and the figures on the left of the subdivisions show how many of those parts are contained in an inch. The construction is as follows:—

Let a given line represent the scale, which is to be divided into a number of equal parts; divide it into as many equal parts as are wanted; this being done, divide the first left-hand division into ten equal parts, which are called subdivisions, each being one-tenth of the primary divisions, and the line will serve as a scale of equal parts.

*Note.*—The numbers are generally reckoned from the left-hand towards the right, beginning at 1, 2, 3, 4, and so on to 10, where the next division is counted 1 again, which denotes it to be 1 above 10. Also the left-hand division is sometimes found to contain 12 parts, for the purpose of taking off feet and inches; in other cases it is divided only into 10, so that decimals can be taken off. We will now explain the use of them.

### *Case 1st.*

When the number to be taken from the scale is less than 100 and not less than 10.

### *Rule.*

Reckon each primary division as 10, and each subdivision as 1. Place one foot of your compasses upon that division, the figure of which denotes the tens which are contained in the given number, and the other foot upon the subdivision, which shows the remaining parts or units, then the given number will be taken from the scale as required.

*Note.*—It must be understood, that the subdivisions stand but for one-tenth of the primary divisions, as has been already observed; so that if each primary division be reckoned as 1, each subdivision must be counted as one tenth; if each primary division be reckoned as 10, each subdivision will then be 1.

### *Examples.*

1st. Let it be required to take 36 from the scale of equal parts.

Put one foot of your compasses upon the 3d division, counting from the left-hand to the right, and the other upon the sixth subdivision, reckoning from the right-hand to the left, which distance will answer to 36 as required.

2d. Suppose it be required to take 85 from the scale of equal parts.

Place one leg of your compasses upon that primary division which has 8 against it, and the other leg upon the fifth subdivision, which will shew the length as required.

### *Examples for Practice.*

1st. Let it be required to lay off 12 from the scale of equal parts.

2d. It is required to lay off 27 from the same scale.

3d. Suppose it be required to lay off 36, 48, and 62, from the scale of equal parts.

4th. Let it be required to lay off 74, 82, and 91, from the same scale.

### *Case 2d.*

When the number to be laid off exceeds 100, and is less than 1000.

### *Rule.*

Here each primary division must be counted as 100, and each subdivision as 10. With one foot of your compasses upon that division whose figure denotes the number of hundreds, extend the other leg to that subdivision which shows the number of tens remaining above the hundreds, and this will be the extent required.

### *Examples.*

1st. Suppose it be required to lay off 240 from the scale.

Place one foot of your compasses upon the second primary division, and the other foot upon the 4th subdivision, which length of your compasses answers to 240, taken from the scale as required.

2d. Let it be required to take 380 from the scale of equal parts.

Put one foot of your compasses upon the division whose figure is 3, and the other upon the 8th subdivision, and this length will be the distance sought.

### *Examples for Practice.*

1st. Let it be required to lay off 280 from the scale of equal parts.

2d. Suppose it be wanted to lay from the scale 240.

3d. It is required to take 460 from the scale.

4th. Suppose it be required to lay off 230 from the same scale.

5th. Let it be required to take 890 from the scale of equal parts.

### *Case 3d.*

To lay off a number not exceeding 10, and not less than 1.

### *Rule.*

In this case each primary division must be reckoned as 1, and each subdivision $\frac{1}{10}$.

Put one foot of your compasses upon that division which expresses the given number, and the other upon that subdivision which shows the parts of an unit contained in the given number, and it will show the distance required. For one-fourth you must take 2½ subdivisions, for one-half 5, and for three-fourths take 7½ subdivisions.

## *Examples.*

1st. It is required to take 3 from the scale of equal parts.

Place one foot of the compasses upon the third division, and extend the other to the beginning of them, which will show the number required.

2d. Let it be required to take 7½ from the scale.

With one foot upon the seventh division, place the other upon the fifth subdivision, which distance will be the extent wanted.

## *Examples for Practice.*

1st. It is required to lay off from the scale of equal parts the number 4.

2d. Suppose it be required to take 5¼ from the scale.

3d. It is required to lay off 6½ from the scale.

4th. Let it be required to take 9¾ from the scale of equal parts.

I might have added several other cases, but the preceding are sufficient for the purpose of understanding their use.

The next scale which comes to our notice is a very important one, and much more useful than any of the former. It is called the diagonal scale. The construction is as follows:

Draw a given line, of any length, which divide into 11 equal parts, being the usual number with which the scale is divided. At the points of division erect perpendiculars, then draw a line parallel to the former, which will form 11 rectangles. Divide each perpendicular into 10 parts, and draw lines through the points of division parallel to the given line; this being done, divide the first division into 10 equal parts, from the left towards the right; also divide the line into 10 equal parts, from the right towards the left; draw lines diagonally from the points of division of one line to the points of division of the other, and the scale is completed. There are two diagonal scales of different lengths, the one being half-an-inch for every division, and the other a quarter.

### *Case 1st.*

To take off a number exceeding two figures, and not more than three.

### *Rule.*

Place one foot of your compasses upon the division, the figure of which denotes the hundreds, and the other upon the subdivision, the figure of which denotes the tens, then count up as many divisions as there are ones in the third figure, place your compasses upon the same division, and subdivision, as they were placed on before, and that distance will answer to the number sought.

### *Examples.*

1st. Let it be required to take 245 from the scale.

Put one foot of your compasses upon the second primary division, which will stand for 200, and extend the other leg to the fourth subdivision, when you will have the distance of 240: now, for the remaining 5, reckon 5 divisions upwards, and place your compasses upon the same divisions as they were before, then the length in your compasses will be the distance for 245 as required.

2d. It is required to take 867 from the diagonal scale.

With one foot of your compasses upon the eighth primary division, place the other upon the sixth subdivision, which will give the distance for 860; afterwards reckon up 7 lines, and place your compasses upon the same divisions, when it will be done.

### *Examples for Practice.*

1st. It is required to take 129 from the diagonal scale.

2d. Let it be required to take off 365 from the same scale.

3d. Suppose it be required to take 742 from the same scale.

4th. It is required to take 998 from the same scale.

### *Case 2d.*

When the number does not exceed 2,000, and not less than 100.

### *Rule.*

In this case you must take from the small diagonal scale, as the large one is divided only into 11 parts; also reckon each primary division as 100, each subdivision as 10, the same as before.

### *Examples.*

1st. Suppose it be required to take 1126 from the small diagonal scale.

Place one point of your compasses upon the eleventh division, and the other part upon the second subdivision; also tell up 6 parallel lines, and place your compasses upon the same divisions as before, which will be the length required.

2d. Let it be required to take 1873 from the same scale.

With one foot of your compasses upon the eighteenth division, extend the other foot to the eleventh subdivision; afterwards tell up 3 parallel lines, and place your compasses upon the same divisions as before, which will give the distance required.

### *Examples for Practice.*

1st. It is required to take 1084 from the small diagonal scale.

2d. Let it be required to take 1374 from the same scale.

3d. Suppose it be required to lay off 1428 from the same scale.

4th. Let it be required to take 1769 from the same scale.

5th. Let it be required to lay off 1984 from the same scale.

### *Case 3d.*

It is required to find the length of a given line.

### *Rule.*

Take the given line in your compasses, and apply it to one of the scales; place one foot upon that primary division, so the other may come amongst the subdivisions; then the divisions upon which your compasses stand will show the length required.

*Note.*—The value of this line depends upon your reckoning the divisions, *i. e.*, whether you count each as 1, 10, or 100. Also, if your compasses will not exactly fall upon a subdivision, tell up as many divisions until they do exactly fall:

### *Examples.*

Let it be required to determine the length of the line A B, (*Fig.* 1st. *Plate* 1st.)

Take the given line in your compasses, and apply it to the scale as directed in the rule, reckon each division as 10, and each subdivision as 1, and the length of the line A B is 212.

### *Examples for Practice.*

1st. It is required to find the length of the line *a* B, (*Fig.* 2d. *Plate* 1st.)

2d. Find the length of the line *b* C, (*Fig.* 3d. *Plate* 1st.)

### *Case 4th.*

Having the length of a line given, to determine the length of another from the same scale.

### *Rule.*

Take the required line in your compasses, and proceed as in the last rule; only remark, that whatever you reckoned each division in the length of the given line, reckon the same in determining the length of the required line.

### *Examples.*

1st. Given the length of the line A B 396; to find the length of the line B C, (*Plate* 1st. *Fig*. 4th.)

Take the line B C in your compasses, and apply it to the same scale from whence A B was determined; observe to place your compasses as directed in the preceding rule, and you will find the length of B C to be 478.

2d. When the length of the line *a b* 1623 is given, to determine the length of the line B C, (*Plate* 1st. *Fig*. 5th.)

The given line of course was taken from the small scale, therefore take B C in your compasses, and place them upon the same scale, as before, and its length is found to be 1246.

### *Examples for Practice.*

1st. Given A B, 482, to find B C, (*Plate* 1st. *Fig*. 6th.)

2d. Required the length of *a b*, when C D 876 is given, (*Plate* 1st. *Fig*. 7th.)

3d. Required the length of the line *b c*, when *c d* 1142 is given, (*Plate* 1st. *Fig*. 8th.)

4th. What is the length of the line *e f*, when *f g* 1896 is given? (*Fig*. 9th. *Plate* 1st.)

---

## DESCRIPTION OF THE LINE OF CHORDS.

This line is very serviceable, the construction of which I shall show, when I come to explain the sector. I shall therefore only give a description and use of it.

This line, then, you will find against the first scale of equal parts, beginning where the other leaves off, and is numbered from the left-hand towards the right. The first degree is counted a 1, and is numbered up to 90 at every tenth degree. I shall now endeavour to shew its use, which is principally to make angles, and determine the measure of them.

### *Case 1st.*

To make an angle which shall contain a given number of degrees.

### *Rule.*

Draw a line of a convenient length, and with the chord of 60 degrees, place one leg of your compasses at one of the extreme points, and describe an arc with the other; then take the proposed number of degrees from the scale of chords, and put one leg upon the point of the line where the arc touches it, afterwards describe an arc with the other to intersect the former, draw a line from the extreme point of the other line through the intersection, and it will form an angle equal to the number of degrees required.

*Note.*—When the angle exceeds 90 degrees, you must take half the given number, and turn your compasses once over upon the arc.

### *Examples.*

Let it be required to make an angle which shall contain 42 degrees.

Draw a line of any convenient length, take the chord of 60, and with one leg upon the line, at the extreme point, describe an arc; then take 42 degrees in the same manner, and apply as directed in the rule; draw the other line, and it will be the angle proposed.

2d. It is required to make an angle which shall contain 108 degrees.

Draw a line, and proceed in the same manner as in the last example, take 54 ($\frac{108}{2}$) and apply to the arc; turn your compasses once over: draw a line from the angular point through the intersection of the arc, and the angle will be finished.

### *Examples for Practice.*

1st. Suppose it be required to make an angle of 29 degrees.

2d. Let it be required to make an angle, which shall contain 12½ degrees.

3d. Suppose it be required to make an angle of 25¾ degrees.

4th. Make an angle that shall contain 48½ degrees.

5th. It is required to make an angle which shall contain 74 degrees.

6th. Let it be required to make an angle of 83½ degrees.

7th. Make an angle of 96½ degrees.

8th. Suppose it be required to make an angle, which shall contain 34 degrees 50 minutes.

9th. It is required to make an angle of 70 degrees.

10th. Make an angle of 138 degrees.

### *Case 2d.*

To find the number of degrees contained in a given angle.

### *Rule.*

1. When the angle is acute take the chord of 60 degrees in your compasses, and with one leg on the angular point, describe an arc with the

other; take that part intercepted between the lines in your compasses, and apply to the scale of chords; place one foot upon the beginning of the said scale, and the division upon which your other leg stands will shew the number of degrees contained in the angle.

2. When the angle is obtuse, *i. e.* when the angle is greater than 90 degrees, take half of the intercepted arc and place upon the scale of chords, and the number of degrees contained therein multiplied by 2 will give the number of degrees in the angle.

### *Example.*

Required the number of degrees contained in the angle A (*Plate* 1st. *Fig.* 10th.)

Take off 60 from the scale of chords place one foot of your compasses upon the angular point, and describe the arc *a b* with the other, which arc take in your compasses, and apply to the scale of chords as directed in the rule, and the number of degrees shown by the division upon which the right leg of the compasses stands is 58.

### *Examples for Practice.*

1st. Show the number of degrees contained in the angle *a*, (*Plate* 1st. *Fig.* 11th.)

2d. Required the number of degrees contained in the angle B, (*Plate* 1st. *Fig.* 12th.)

3d. How many degrees are in the angle D? (*Plate* 1st. *Fig.* 13th.)

4th. Find the number of degrees in the angle E, (*Plate* 1st. *Fig.* 14th.)

*Observe*—As the scale is divided only into degrees, minutes therefore cannot be taken from it with any considerable accuracy.

There are two other scales of equal parts, on the same face as the diagonal scale; the construction and use being the same as the preceding, it will be superfluous for me to illustrate them.

## PARALLEL RULER.

The parallel ruler is a very useful instrument, and is used to draw lines parallel; it has two parts, connected to each other by two brass bars of equal length, fastened to each part by screws. I will explain the utility of it as briefly as possible.

### *Case* 1*st.*

To draw a line from a given point parallel to a given line.

### *Rule.*

1st. When the given point is not farther from the given line than the ruler will extend, lay the edge of your ruler upon the given line, then extend it until the other edge lies upon the given point, taking care that it be not moved from the given line; draw a line through the said point, and it will be parallel as required.

2d. When the given point is further situated from the given line than your ruler will extend, lay your ruler as before directed, and with the greatest extension draw a line upon that edge of the ruler which does not lie upon the given line, upon which place that edge of the ruler which laid upon the given line, and with the greatest extension draw a line upon another edge; proceed in the same manner until the edge will reach the given point, from which draw a line as mentioned above, and it will be done.

### *Examples.*

1st. Let it be required to draw a line from the given point A parallel to the given line B C, (*Plate* 1st. *Fig*. 15th.)

Place one edge of your ruler upon B C, and open it until one of the other edges comes up to the given point A, then draw the line A D, and it will be parallel to B C as required.

2d. It is required to draw a line from the given point B, parallel to the line A C, (*Plate* 1st. *Fig*. 16th.)

Lay the edge of your ruler as in the last example, and extend it as far as you are able, whence draw the line *a b* upon the farthest edge, then lay the same edge (that laid upon the given line) on *a b* and open the ruler until one of the edges comes up to the given point; draw the line B D, and it will be parallel to A C, as required.

### *Examples for Practice.*

1st. Let it be required to draw a line parallel to the given line C D, from the given point B, (*Plate* 1st. *Fig*. 17th.)

2d. Let it be required to draw from the given point *e*, a line parallel to the given line *c d*, (*Plate* 1st. *Fig*. 18th.)

3d. It is required to draw a line parallel to the given line E F, from the given point D, (*Plate* 1st. *Fig*. 19th.)

### *Case 2d.*

To determine whether a line is parallel to a given line.

### *Rule.*

Lay the edge of your ruler upon the given line, and open it by the last rule, until one of the other edges touches the other line; then, if the edge lies upon the whole line, it is drawn parallel; but, if it lies only on a part of the line, then it is not drawn parallel, but inclines to the given line; and if it be produced, it will somewhere touch it.

---

## DESCRIPTION AND USE OF THE PROTRACTOR

The protractor is made of brass, its diameter is usually about 3¾ inches; the ring is divided into 180 parts, each part containing 1 degree, and is reckoned both from the right to the left, as well as from the left to the right; it is also numbered at every tenth degree both ways, for the sake of convenience, and the diameter has a small notch in the middle, which denotes the centre, and serves for various purposes, which will be subsequently explained.

### *Case* 1*st.*

To erect a perpendicular to a given line, from a given point.

### *Rule.*

Place the diameter exactly upon the line, and the notch against the point from whence the perpendicular is to be erected; then make a mark upon the paper with a pencil against the ninetieth degree, draw a line from the given point through the said mark, and it will be the perpendicular required.

### *Examples.*

1st. Let it be required to erect a perpendicular from the given point, *b* in the line A B, (*Plate* 1st. *Fig*. 20th.)

Put the diameter of the protractor upon the given line A B, and the notch against the point *b*, mark as directed in the rule the ninetieth degree on the paper; draw the line *b* C, from *b*, through C, and *b* C will be the perpendicular sought.

2d. It is required to erect a perpendicular upon the given line C D, from the given point 0, (*Plate* 1st. *Fig*. 21st.)

Put the diameter as before, and the notch against the point *o*; observe the ninetieth division, and draw the line *o* E through both points, which will be the perpendicular wanted.

### *Examples for Practice.*

1st. It is required to make a perpendicular from the given point *a*, in the given line A B, (*Plate* 1st. *Fig*. 22d.)

2d. Let it be required to erect a perpendicular upon the given line C D, from the given point *b*, (*Plate* 1st. *Fig*. 23.)

### *Case 2d.*

To find the number of degrees contained in any given angle.

### *Rule.*

Place the diameter of the protractor upon one of the lines which forms the angle, and the notch at the angular point, then observe what division stands on the other line, which will show the number of degrees contained in the angle as required.

### *Examples.*

1st. Find how many degrees are contained in the angle A, (*Plate* 1st. *Fig.* 24th.)

Lay the diameter upon one line, and the notch at A, and the other line will be observed to pass under the forty-seventh division; wherefore the number of degrees contained in the angle A is 47.

2d. Let it be required to find the number of degrees contained in the angle B, (*Plate* 1st. *Fig.* 25th.)

Place the diameter upon either of the lines, and the notch upon the angular point as before, and it will be found that the other line passes under the 108th degree; the number therefore contained in the angle is 108.

*Note.*—In finding the number of degrees contained in any angle, you must observe the division you reckon, *i. e.* you must not confuse the right-hand division with the left-hand.

### *Examples for Practice.*

1st. Let it be required to find the number of degrees contained in the angle C, (*Plate* 1st. *Fig.* 26th.)

2d. Required the number of degrees contained in the given angle D, (*Plate* 1st. *Fig.* 27th.)

3d. Required the number of degrees contained in the angle E, (*Plate* 1st. *Fig.* 28th.)

4th. Let it be required to find the number of degrees contained in the angle F, (*Plate* 1st. *Fig.* 29th.)

5th. Let it be required to find the number of degrees contained in the angle G, (*Plate* 1st. *Fig.* 30th.)

### *Case 3d.*

To lay off angles, which shall contain a given number of degrees.

### *Rule.*

Draw a line of any length; place the diameter of the protractor upon it, and the notch against the angular point; make a mark against that division of the protractor which denotes the number of degrees in the angle; draw a line from the angular point through the said mark, and it will form an angle, which shall contain the number of degrees proposed.

### *Examples.*

1st. Lay down an angle that shall contain 38 degrees.

Draw an indefinite line; lay the diameter upon it, and the middle or notch upon the angular point, which may be wherever you please; then against the thirty-eighth division make a mark, draw a line from the angular point through the mark, and it will make an angle with the other line of 38 degrees as required.

2d. Let it be required to make an angle of 106 degrees.

Draw a line as in the last example, place the diameter and notch as before, make a mark against the hundred-and-sixth division, which denotes the number of degrees proposed; draw a line through both points, and it will make an angle with the other line of 106 degrees as required.

*Note.*—In marking off the number of degrees; the right or left division may be used as is most convenient.

### *Examples for Practice.*

1st. Let it be required to make an angle of - - - - - 29 degrees
2d. Another - - - - of 81 ditto.
3d. Another - - - - of 64 ditto.
4th. Another - - - - of 71 ditto.
5th. Another - - - - of 96 ditto.
6th. Another - - - - of 132 ditto.
7th. Another - - - - of 89 ditto.
8th. Another - - - - of 142 ditto.
9th. Another - - - - of 42 ditto.
10th. Another - - - - of 172 ditto.

## DESCRIPTION AND USE OF THE SECTOR.

THE sector consists of two equal legs turning about a center, from which are drawn several lines called sectorial lines; there are other lines also drawn parallel to the edge of the sector, like those upon a Gunter's scale. The lines on both faces of the instrument are represented as they are found on the sector.

I shall first give a description and use of those lines which are called sectorial, and afterwards explain the use of those drawn parallel to the sector.

Before I begin, it may be proper to observe that the sectorial lines are drawn once on each leg of the same face of the instrument.

The sector consists of seven sectorial lines, which are called as follows: on one face are 1st. a line of sines on each leg marked *Sin.* or *S.* 2d. A line of tangents on each leg, marked *Tan* or *T.* 3d. Another line of tangents marked

*T* or *t*, extending from 45 to about 75 degrees. On the other face are 4 more lines: 1st. a line of equal parts marked L I N, or L. 2d. The line of chords marked C H O, or C. 3d. A line of secants marked SEC, or S; and 4th. A line of polygons marked POL.

All the sectorial lines are numbered from the center, except the line of polygons, which is reckoned from the right-hand towards the left. The line of sines extends as far as 90. The line of tangents, to the large radius as far as 45, and the line of tangents to the less radius, as has been before-mentioned, to 75 degrees. The line of equal parts on the other face is divided into 10 parts, and each sub-divided into 10 more. The line of chords extends as far as 60 degrees, the radius of a circle; the line of secants to about 75 degrees; and the line of polygons from 4 (beginning at 10 on the line of equal parts) to 12.

The distance between 10, and 10 on the line of equal parts, is the same as the distance between 60, and 60 on the line of chords; 45, and 45 on the line of tangents, and likewise between 90, and 90 on the line of sines.

I shall now explain in as brief and simple a manner as I can, the method of constructing the lines of Chords, Sines, Tangents, and Secants.

*Note.*—The construction of the scale of equal parts is the same as has been before explained on the plane scale.

### *To construct Lines of Sines, &c.*

With any convenient radius, describe the circle A, D, B, C, see *Fig.* 1st. *Plate* 2d. Draw the diameters A B; C D intersecting each other at right angles; produce the diameter C D, at pleasure, to E, and at B erect the perpendicular B F. Draw the line B D, and divide the quadrant B D into 9 equal parts, each containing 10 degrees. Take your compasses, and place one foot at B, then with the radii B 10, B 20, B 30, &c. describe arcs upon the line B D, and B D will be a line of chords; the figures 10, 20, 30, &c. on the line B D, denoting the chords of 10, 20, 30 degrees, &c. in the quadrant B D, which produce until they meet the line B F, then the perpendicular B F will be a line of tangents.

From the points 10, 20, 30, &c. in the quadrant, let fall the perpendiculars upon the diameter A B.

The lines O 10, O 20, O 30, &c., drawn from the center O to the points 10, 20, 30, &c. on the perpendicular B F are secants.

Having thus shown the manner of constructing those lines drawn on a sector, which are termed sectorial, I shall now proceed to show the use of them:—by first laying down the rules in as explicit a manner as I can; illustrating each case by a number of examples, and afterwards inserting a number of miscellaneous questions for practice.

*Note.*—It may be proper to remark, that in using the sector, we can accommodate ourselves with any radius we please; every part, or side, being always proportional, as founded on the property of similar triangles. Also observe, that the measure taken from the center to any point on a sectorial line, is called a lateral distance; and the measure from any point on a sectorial line, from one leg to its corresponding point on the other is called a transverse distance.

## USE OF THE LINE OF EQUAL PARTS.

### *Case* 1*st.*

To divide a line into any number of equal parts.

### *Rule.*

Take the given line in your compasses, and place one foot on the figure which denotes the number of parts the line is to be divided into, and extend the other to the same figure on the other leg: then the transverse distance from 1 to 1 will be one of those equal parts required; or, if the line should be longer than can be applied to the sector, take some aliquot part or parts of it, and multiply the transverse distance taken from the sector by the said aliquot part or parts.

### *Examples.*

1st. Let it be required to divide the line A B, (*Plate* 2d. *Fig*. 2d.) into 7 equal parts.

Take the given line in your compasses, and make it a transverse distance from 7 to 7, as directed in the rule; then take the transverse distance from 1 to 1, and apply it seven times on the line.

2d. Let it be required to divide the line B C into 12 equal parts, (*Plate* 2d. *Fig*. 3d.)

*Note.*—If the number of parts exceeds 10, reckon each division as 10; if the number exceeds 100, reckon it as 100; and so on.

Take $\frac{1}{2}$ the line C D in your compasses, and make a transverse distance from 12 to 12, and proceed as before; take care to double the transverse distance from 1 to 1, for the twelfth part of the line C D.

### *Examples for Practice.*

1st. Divide the given line, *a b*, (*Plate* 2d. *Fig*. 4th.) into 8 equal parts.

2d. Divide the given line *b c*, into 5 equal parts, also the line *c d* into 9 equal parts (*Plate* 2d. *Fig*. 5th.)

### *Case 2d.*

To divide a given line into any number of parts, that shall be in any given proportion.

### *Rule.*

Take the given line in your compasses, and make it a transverse distance equal to the sum of the proposed numbers; then take the transverse distances of the above-mentioned numbers, and they will be the parts required.

### *Example.*

Let it be required to divide the line A B into 3 parts, in the proportion of 1, 2, 3, (*Plate* 2d. *Fig*. 6th.)

Take the given line, and apply it as a transverse distance from 6 to 6, (the sum of the numbers 1, 2, and 3;) then take the transverse distances of the numbers 1, 2, and 3, respectively; apply them to the given line, and it will be divided as required.

### *Examples for Practice.*

Divide 3 given lines, each in the proportion of 2, 3, and 5; 4 others in the proportion of 7, 8, 9, and 12; and seven others in the proportion of 6, 8, 11, 13, and 14.

### *Case 3d.*

When a given line is divided into any number of parts which are equal, and it is required to find how many of those parts are contained in any other line.

### *Rule.*

Take the given line in your compasses, and make it a transverse distance; by placing one foot upon the figure, which denotes the number of parts, and the other point to its corresponding figure on the other leg; then with the same opening of the sector take the other line in your compasses, and apply it transversely in the same manner; so that both points may fall on corresponding divisions, and these points will show the number of parts required.

### *Example.*

Suppose the given line *a b* is divided into 9 parts; it is required to find how many of those parts are contained in the line A B, (*Plate* 2d. *Fig.* 7th.)

With the line *a b* in your compasses, and one foot upon 9, open the sector, and lay your compasses upon the other 9; then take the other line A B, and (with the same opening of the sector) apply it as before, and the points will show the number of required parts to be 6.

### *Examples for Practice.*

1st. Given a line divided into 7 parts, find how many of those parts are contained in another given line.

2d. Suppose the side of a square be 12, what is the side of another square, given by position?

3d. Given the side of a triangle 40; to find the other sides, the triangle being given by construction.

### *Case 4th.*

To find a fourth proportional to three given numbers or lines.

### *Rule.*

With the lateral distance of the second number or line, apply it as a transverse distance of the first; then the transverse distance of the third number or line, laid off from the center as a lateral distance, will be the fourth proportional required.

### *Examples.*

1st. Given three numbers 4, 5, and 8, to find a fourth proportional.

Take 5 as a lateral distance in your compasses, and make it a transverse distance of 4; then take the transverse distance of 8, and place one of the points of your compasses upon the center, and the other (as a lateral distance) upon the line of equal parts, and it will extend to 10, the fourth proportional required.

2d. Given the three sides of a triangle, 16, 14, and 11; to find two sides of another triangle, the other side being 9 and the triangles similar.

By properly considering this question, it evidently appears that the statings are these:

16 : 14 : : 9 to one side,
and 16 : 11 : : 9 to the other side;

where a fourth proportional may be found to each stating, as in the first example, which will be each side required.

### *Examples for Practice.*

1st. Given 3 numbers 9, 13, and 18, to find a fourth proportional.

2d. Given 3 lines A, B, C, to find a fourth proportional, (*Plate* 2d. *Fig.* 8th.)

3d. Suppose the length of a rectangle be 18, and its breadth 13; required the length of another rectangle, whose breadth is 8.

### *Case 5th.*

When a given line is divided into a number of parts in a certain proportion (as in case 2d.) and it is required to divide another given line in the same proportion.

### *Rule.*

Take the first given line, and make it a transverse distance from 10 to 10; then take the given parts in your compasses, and apply them transversely, observing what divisions your compasses stand on; afterwards take the other line, and make the transverse distance from 10 equal to it, and take the above-mentioned divisions from the sector; apply them to the line, which will then be divided in the same proportion as the former line.

### *Example.*

Let it be required to divide the line B C into 4 parts, when the given line A D is divided in the same proportion: namely, A 1; 1, 2; 2, 3; 3 D, (*Plate* 2d. *Fig.* 9th.)

Take A D in your compasses, and apply it transversely from 10 to 10; also take the parts A 1, 1, 2, &c., and put on the sector, observing what divisions the points stand on; then take B C, and apply it transversely as before, from 10 to 10; afterwards take off the above-mentioned divisions, and apply from B to C; whence B C will be divided as required.

### *Examples for Practice.*

Divide three lines, of different lengths, in the same proportion as the line A D, (*Plate* 2d. *Fig.* 10th.)

Divide two lines into nine parts, in the same proportion as A a, (*Plate* 2d. *Fig.* 11th.)

Having thus shewn the utility of this line, by applying it to some of the cases in general use, I shall now explain the scale of chords, so as to give, in as clear a manner as possible, a concise view of the subject.

## SCALE OF CHORDS.

### *Case* 1*st.*

To make angles of any proposed number of degrees.

### *Rule.*

Take transversely the chord of 60 degrees at any convenient opening of the sector, and describe an arc; then take the transverse distance of the proposed number of degrees, and place both points of your compasses upon the arc, draw lines from the center through the two said points, which will form the angle required.

### *Examples.*

1st. It is required to make an angle of 40 degrees.

With the chord of 60 degrees taken by Rule, describe an arc; also take 40 degrees in the same manner, and lay off upon the arc; draw lines from the center through the extremities of the said arc, and these lines will form the angle sought.

2d. It is required to lay down an angle of 38° 12′.

Describe an arc with the chord of 60° as before; also take 38° 12′, and apply it in the same manner; draw lines from the center as abovementioned, and it is done.

3d. Let it be required to make a triangle, whose 3 angles are 70, 54, and 56 degrees.

Draw a line of any length, and at one end lay off the angle of 70 degrees as before directed; also at the other end lay off the angle of 54 degrees; draw lines from the two ends, and their intersection will form the triangle whose three angles will be 70, 54, and 56, as required.

*Note.*—It must be remembered, that when the angle exceeds 60 degrees, a half or a quarter of it must be taken, and the compasses turned once or twice over.

### *Examples for Practice.*

1st. It is required to make an angle of 31 degrees.

2d. Make an angle that shall contain 78 degrees.

3d. Make an angle of 52° 33′.

4th. It is required to make a triangle whose three angles are 39, 85, and 56 degrees.

5th. Make an angle that shall contain 105 degrees.

6th. Let it be required to make a triangle whose vertical angle shall be 74 degrees, and the other two angles equal.*

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* When two angles of any triangle are equal, they may be found by subtracting the other from 180 degrees, and taking half the difference.

### *Case 2d.*

To find how many degrees are contained in any angle.

### *Rule.*

With the chord of 60 degrees taken as directed in the last rule, and one foot of your compasses upon the angular point, describe an arc from one line to the other; then take the length of this arc in your compasses, and apply it transversely upon corresponding divisions, which divisions will show how many degrees are contained in the angle.

### *Examples.*

1st. Find how many degrees there are in the angle A, (*Plate* 2d. *Fig.* 12th.) With the chord of 60 degrees, and one foot of your compasses on A, describe the arc *b c*, take the length *b c*, in your compasses, and apply it transversely, when the points will be found to stand on the fortieth division, and consequently the measure of the angle A is 40 degrees.

2d. How many degrees are contained in each angle of the triangle A, B, C? (*Plate* 2d. *Fig.* 13th.)

Take the chord of 60 degrees as in the last example, and at each angular point A, B, and C, describe the arcs *a b*, *c d*, *e f*, respectively; then, with the length of each intercepted arc applied according to rule, angle A will be found to be 54° 16′; angle B 89° 38′, and angle C 36° 6′.

### *Examples for Practice.*

1st. How many degrees are contained in the angle A? (*Plate* 2d. *Fig.* 14th.)

2d. Find the measure of the obtuse angle B, (*Plate* 2d. *Fig.* 15th.)

3d. Also the number of degrees in the angle C, (*Plate* 2d. *Fig.* 16th.)

4th. How many degrees are there in each angle of the triangle A, B, C? (*Plate* 2d. *Fig.* 17th.)

5th. Required the number of degrees in each angle of the figure A, B, C, D, (*Plate* 2d. *Fig.* 18th.)

*Note.*—The number of degrees in any part of the circumference of a circle may be found by merely taking the radius of the circle in your compasses, and fitting it transversely to the sector from 60 to 60; then take the given arc or circumference, and apply it in the same manner, and the divisions, which the points of your compasses stand on, will show the number of degrees in the arc.

## USE OF THE LINE OF SINES.

A right sine is a line drawn from the extremity of any arc of a circle perpendicular to the diameter, which is drawn from the other extremity of the arc.

### *Rule* 1*st*.

When radius and the degrees of the arc are given to find the length of the sine.

Take the length of the radius in your compasses, and make it a transverse distance from 90 to 90 on the line of sines; then take the number of degrees of the arc in your compasses from the same line, and apply it to the scale from whence the length of the radius was taken, which will show the length of the sine required.

### *Example.*

Given the radius 5 inches, and the degrees of the arc 40; required the length of the sine.

Take 5 inches in your compasses, and set it transversely from 90 to 90; then, with the same opening of the sector, take 40 degrees from it, and set on the scale of inches, whence the length of the sine is found to be 3, 2 inches.

### *Examples for Practice.*

1st. What is the length of the sine of an arc that contains 43 degrees, whose radius is 8 inches?

2d. Shew the length of the sine, the arc of which is 84 degrees, and diameter 12 inches.

3d. Required the length of the sine, the arc of which is 63 degrees, and radius 4 inches.

4th. What is the length of the sine of an arc, whose radius is an inch, and degrees of the arc 59?

### *Rule 2d.*

When the lengths of the sine and radius are given, to find the number of degrees contained in the sine.

With the length of the radius in your compasses, apply it transversely from 90 to 90, (always remember to have your sector opened equal to your radius,) take the length of the sine, and put it transversely on the line; take care to place it upon corresponding divisions, then the divisions, which your compasses stand upon, will show the number of degrees in the sine required.

### *Examples.*

1st. What is the number of degrees in the sine, whose length is 4 inches, and radius of the circle 7 inches?

Take 7 inches in your compasses, and apply it transversely from 90 to 90 according to rule; then take 4 inches, and place upon the sector as directed in the rule, and the degrees of the sine will be found to be 36¾.

2d. How many degrees are there in the sine, whose length is 5 inches, and radius of the circle 6?

With 6 inches in your compasses, put them from 90 to 90 on the line as in the last example; afterwards take 5 inches, and put on the line, and your compasses will stand on the 57th division; wherefore the number of degrees in the sine is 57.

### *Examples for Practice.*

1st. What is the number of degrees contained in the sine, whose length is 3½ inches, and length of the radius of the circle 4?

2d. How many degrees are there in the sine, whose length is 3¾ inches, and diameter of the circle 15¾?

3d. How many degrees are contained in the sine, the length of which is 4¼ inches, and diameter 14?

4th. Required the number of degrees in the sine, the length of which is 4½ inches, and radius of the circle 16⅜?

## USE OF THE LINE OF TANGENTS.

I shall again refer to the trigonometrical canon, where it may be observed that a tangent is a line drawn to touch the circumference of a circle without cutting it, as B F. I shall now proceed to explain the use of this line in the following manner.

### *Case 1st.*

How to take off any number of degrees.

### *Rule.*

1st. When the proposed number is less than 45 degrees.

Open your sector to any convenient radius, place one foot of your compasses upon that division on the large radius, which denotes the number of degrees proposed, and the other foot upon the same division on the other leg of the sector, and it will be done.

2d. When the given number exceeds 45 degrees.—Place your compasses on the less radius, and proceed as before.

### *Examples.*

1st. Take 24 degrees from the line of tangents.

Take your compasses and place one foot on the 24th division, and the other on the corresponding division on the other leg, which will give the tangent required.

2d. Take 63 degrees from the line of tangents.

Place one foot of your compasses on the 64th division on the less radius, and the other upon the same division on the other leg, which will give the tangent required.

### *Examples for Practice.*

1st. Let it be required to take off 28 degrees.
2d. It is required to take off 39 degrees.
3d. It is required to take off 72 degrees.

### *Case 2d.*

Having the length of the radius, and degrees of the arc given, to find the length of the tangent.

### *Rule.*

With the length of the radius in your compasses, make it a transverse distance from 45 to 45 on the large, or from 45 to 45 on the less radius; then take transversely the number of degrees of the arc from corresponding divisions, and apply it to the scale from whence you took the length of the radius, which will shew the length of the tangent.

### *Examples.*

1st. Given the length of the radius 2 inches, and the degree of the arc 41, what is the length of the tangent?

Take 2 inches, and apply it from 45 to 45 as your rule directs; then take 41 degrees from the line, and apply it to the scale of inches; whence the length of the tangent will be found to be 1$\frac{3}{4}$.

2d. Required the length of the tangent when the degrees of the arc are 38, and the length of the radius 3$\frac{3}{4}$ inches.

With the given radius 3$\frac{3}{4}$ inches in your compasses, apply it transversely from 45 to 45; afterwards take 38 from the line, as has been before explained, and place it on the scale of inches; where the length of the tangent will be found equal 2,97 inches.

3d. What is the length of the tangent, when the radius is 1$\frac{3}{4}$ inches, and degrees of the arc 65?

Take 1$\frac{3}{4}$ inches, and put it transversely from 45 to 45 on the less radius, which you will find against 14 on the greater; then take 65 from the said line, taking care to place your compasses on corresponding divisions, which distance apply to the scale of inches, and you will find the length of the tangent to be 3¾ inches.

### *Examples for Practice.*

1st. Required the length of the tangent, when the degrees of the arc to which the tangent belongs are 24, and the length of the radius 2¼ inches.

2d. Required the length of the tangent, when the degrees of the arc are 54, and the length of the radius is 2½ inches.

3d. What is the length of the tangent, if the radius be 13½ inches, and the number of degrees of the arc are 33?

4th. If the number of degrees of the arc be 73, and radius 4¼ inches, what is the length of the tangent?

### *Case 3d.*

When the lengths of the radius and tangent are given, to find the number of degrees contained in the arc.

### *Rule.*

Take the length of the radius, and place it transversely from 45 to 45, as in the last rule; afterwards take the length of the tangent in your compasses, and place it upon the same line, observing as before that your compasses stand upon corresponding divisions, which will shew the number of degrees required.

*Note.*—When the length of the tangent exceeds radius, the degrees of the arc will be more than 45; therefore place the radius from 45 to 45 on the less of the two said radii.

### *Examples.*

1st. Given the length of the tangent 3 inches, and the length of the radius $6\frac{1}{2}$ inches; what are the number of degrees contained in the arc to which the tangent belongs?

With $6\frac{1}{2}$ inches in your compasses, make it a transverse distance from 45 to 45; then take three inches (the length of the tangent) which apply to the line, your sector preserving the same radius, and the degrees answering thereto will be $26\frac{1}{2}$.

2d. What is the number of degrees contained in the arc whose tangent is 6 inches, and the length of the radius 3?

Take 3 inches in your compasses, and place it from 45 to 45 on the less radius, afterwards take 6 inches in your compasses, and place it transversely upon the line, and the number of degrees answering thereto will be about 63.

### *Examples for Practice.*

1st. What is the number of degrees contained in the arc, the tangent of which is 6 inches, and radius $3\frac{1}{4}$?

2d. Required the degrees of the arc whose radius is $2\frac{3}{4}$ inches, and length of the tangent $4\frac{3}{4}$.

3d. Shew the number of degrees contained in the arc whose tangent is 5 inches, and radius of the circle $2\frac{1}{4}$.

## USE OF THE LINE OF SECANTS.

The use of this line is similar to the line of tangents; the cases, and the method of solving them, being the same: it will therefore be needless for me to dwell upon it. I shall only observe, that for finding the length of a secant to a given radius and arc; or, on the contrary, for finding the arc to a given radius and secant, it will be best to refer to the rules laid down for the use of the line of tangents; the only difference is, that instead of extending the compasses from 45 to 45, extend them from 75 to 75.

I have now briefly given the use of each line; but the use of one when not dependent on any other is very confined; for by the relation of them we are able to determine sides, and angles of triangles, besides many other things.

### *Case 1st.*

When the hypothenuse and one of the acute angles are given, to find the base and perpendicular.

### *Rule.*

Say, as radius is to hypothenuse, so is the verticle angle to the base, and so is the cosine of that angle to the perpendicular; that is, take the hypothenuse and apply it from 90 to 90 on the line of sines; then take the vertical angle from the line, *(Rule 1st. on the use of this line)*, and apply it to the scale from whence the hypothenuse was taken, which will show the base; afterwards take the cosine of the same angle, and proceed in the same manner, which will be the perpendicular.

### *Example.*

Given the hypothenuse 370, and the angle opposite the base 33° 30′, to find the base and perpendicular.

Take from some convenient rule or scale of equal parts 370, which make a transverse distance from 90 to 90; then take as mentioned in the rule 33° 30′, and apply to the above-named scale, when the perpendicular will be found equal to 204,2; afterwards take 56° 30′ from the line of sines, and apply to the scale, and the perpendicular will be 308.

*Note.*—The lateral distances can be taken from the line of equal parts, already treated of.

### *Examples for Practice.*

1st. What are the base and perpendicular, when the hypothenuse is 295, and one of the angles 38°?

2d. Required the base and perpendicular, when one of the angles is 84, and hypothenuse 842.

3d. Shew the base and perpendicular, if the hypothenuse be 629, and one of the angles 82° 30′.

*Note.*—In any right-angled triangle, when one of the acute angles is known, the other may be found by subtracting it from 90 degrees.

### *Case 2d.*

When the angles and one of the sides are given, to find the hypothenuse and other side.

### *Rule.*

Take the given side in your compasses, and place it on the degree which denotes the angle opposite the given side; then extend your compasses from 90 to 90, and place them as a lateral distance on the line of equal parts, which will be the hypothenuse. Again take the other angle from the line of sines, and proceed as before, which will give the other side.

### *Example.*

Given the angles of a triangle, one 34° 24′, the other 55° 36′, and the base 280, to find the perpendicular and hypothenuse. With 280 in your compasses, place them from 55° 36′ to 55° 36′ (the angle opposite the base) on the line of sines; then take the distance from 90 to 90, and make it a lateral distance on the line of equal parts, which will be the hypothenuse 340; again take the other angle 34° 24′, and the perpendicular will be found to be 191.

### *Examples for Practice.*

1st. When the base is 97, and the angles 24°, 34′ and 65° 26′, required the hypothenuse and perpendicular.

2d. Required the hypothenuse and perpendicular, if the base be 360, and the angles 36° 12′ and 53° 48′.

3d. If a ship sails N. 63° W. until her departure becomes 83 miles; required the distance sailed, and difference of latitude.

### *Case 3d.*

The hypothenuse, and one of the sides given, to find the angles and other side.

### *Rule.*

Take the hypothenuse in your compasses, make it a transverse distance from 90 to 90 on the line of sines: afterwards take the given side in your compasses, and place it transversely on the same line; then the division upon which your compasses stand, will show the angle opposite the given side, which subtract from 90, and you will have the other angle. Take this angle transversely from the line of sines in the same manner as the preceding, and make it a lateral distance on the line of equal parts, or upon that scale from whence you took the other sides, and it will give the required side.

### *Example.*

Given the hypothenuse 324, and the base 260; required the angles and perpendicular.

Take 324 in your compasses, and make it a transverse distance from 90 to 90 on the line of sines, take in the same manner 260 and place upon the said line, and the opposite angle will be found to be about 55 degrees, which subtract from 90, and you will have the other angle 35 degrees; then take 35 in your compasses, and apply to the scale from whence you took the other sides or lines, which will show the perpendicular to be 183.

### *Examples for Practice.*

1st. Required the angles, and perpendicular of a triangle, when the base is 96, and hypothenuse 195.

2d. When the base is 245, and hypothenuse 473, required the angles and perpendicular.

3d. Required the course and departure of a ship which sails south-easterly 214 miles, and her difference of latitude 112.

### *Case 4th.*

When the base and perpendicular are given, to find the angles and hypothenuse.—

Take the perpendicular, and apply it transversely on the line of sines from 90 to 90; also take the base in your compasses, and put it transversely on the line of tangents; then the degree upon which your compasses stand will be the angle opposite the base; subtract it from 90, and you will have the other angle. Take the perpendicular, and apply it transversely upon the degree which denotes the last-mentioned angle on the line of sines, then the extent from 90 to 90 applied to the same scale from whence you took the other lines, and you will have the hypothenuse.

### *Example.*

Required the hypothenuse, and angles of a triangle, when the base is 216 and perpendicular 321.

Set the perpendicular 321 from 90 to 90 on the line of sines; again take 216, and place it transversely on the line of tangents, and the degree upon which your compasses stand will show the number of degrees contained in the angle opposite the base, which is 34; subtract it from 90, and it will give the other angle 56 degrees. Take 216 and apply it from 34 to 34 on the line of sines; then take the extent from 90 to 90, and apply it to the scale of equal parts, when the hypothenuse will be found to be 428.

*Note.*—When the base is greater than the perpendicular, find the angle opposite the latter line first; and use the base as the perpendicular.

### *Examples for Practice.*

1st. Given the base 98, and perpendicular 211; required the hypothenuse and angles.

2d. Required the hypothenuse and angles of a triangle when the base is 235, and perpendicular 38.

3d. Required the hypothenuse and angles of the triangle whose base is 269, and perpendicular 136.

### *Case 6th.*

Having a side, and the angles; or an opposite angle, and two sides of any oblique-angled triangle; to find the other parts.

### *Rule.*

Take the given side from a scale of equal parts, and place your compasses upon that degree on the line of sines which denotes the angle opposite the given side; afterwards take the next angle from the said line, and put it upon the scale of equal parts, which will shew the side opposite the angle; in the same manner take the angle opposite the other side from the line of sines, and apply it to the scale of equal parts, which will give the said side. When an angle and two sides are given, take the line opposite the given angle, and place your compasses upon the degree which denotes the given angle on the line of sines; then take the other given side and apply it transversely on the line of sines, and the degree which your compasses stand on will shew the angle opposite to it; in in like manner proceed with the other angles, and the required line may be found as above.

## *Examples.*

1st. Given one side of a triangle 96, and the angle opposite 48, the other two angles 96 and 36; required the other two sides. Take 96 in your compasses, and place from 48 to 48 on the line of sines; afterwards take 96 degrees (your sector keeping the same radius) from the line, and apply on the scale of equal parts, and the line opposite will be found to be 129; then with the angle opposite the other given side taken from the line of sines applied to the scale of equal parts, the said side will be 77.

*Note.*—When any angle is greater than 90, subtract it from 180, and take the sine of the remainder.

2d. Given a side of a triangle 236, and the angle opposite 87°: also another side 174; required the other side and angles.

With 236 in your compasses, apply it from 87 to 87 on the line of sines; also with 174 in your compasses place them upon the line of sines as directed in the rule, and they will be found to stand on the thirty-second division, wherefore the number of degrees contained in the angle opposite the side 174 is 32, and the other angle of course is 61; also the remaining side will be found, as in the last example, to be 208.

## *Examples for Practice.*

1st. Given the side of a triangle 3, 16, and the angle opposite 65 degrees, also the two remaining angles 48 and 67; what are the other two sides?

2d. Shew the angles and side of a triangle, which has two sides 260 and 340, and an angle opposite the former 63 degrees.

3d. Required the distance of a windmill from an observer, where the angle between it and a tree was 62 degrees; the tree being 96 yards from the place of observation, and 86 from the windmill.

Having thus concisely shown the use of the sectorial lines in the solution of triangles, I will next explain those lines which are drawn parallel to the edge of the sector.

## LINE OF NUMBERS.

As this is the first that comes to my notice, being the most important, I intend to explain the manner in which it is divided, and afterwards the use of it. The division then begins at the left hand, and is numbered from 1 to 10 in the middle; each division is divided into 6 parts; from 10 in the middle it is numbered up to 100 towards the right hand; the numbers are 20, 30, &c. being ten times the value of those on the left of the middle 10—each division also is subdivided into 10 parts, and these again into 2 parts. The value of the first divisions governs that of the subdivisions, for if we reckon the first divisions in the order of 1, 2, 3, &c. to 10, and from 10 to 20, 30, 40, &c., each subdivision on the left of the middle 10 will be equal to $\frac{1}{10}$ of an unit; those on the right equal to 1 or unit, and the second subdivision $\frac{1}{2}$ or half an unit; or if the unit on the left hand stand for 10, then it will be reckoned 10, 20, 30, &c. up to 100 in the middle, and the numbers on the right hand will be 200, 300, &c., each subdivision reckoned on the left hand is 1, and on the right hand each subdivision is 10, 20, 30, &c. and its subdivision is 5. Again, if the figure on the left hand represents $\frac{1}{10}$, then the next division will stand for $\frac{2}{10}$, &c. The 10 in the middle will stand for 1, the 2 on the right hand for 2, &c. After giving a description of this line, I shall shew its use in the following manner.

### *Case* 1*st.*

To multiply two numbers together.

### *Rule.*

Extend your compasses from unit to the multiplicand, and that distance will reach from the multiplier to the product.

*Note.*—The multiplicand is the number multiplied; the number multiplied by is the multiplier, and the product is the number arising from the multiplication of the multiplicand and multiplier.

### *Examples.*

1st. Let it be required to multiply 10 by 4.

Set one foot of your compasses upon 1, and the other upon 10; and that extent will reach from 4 to 40, the product required.

2d. Suppose it be required to multiply 80 by 36.

In this example you must reckon each chief division as 10, therefore extend your compasses from 1 to 8; then, with that extent, place one point of your compasses upon the sixth subdivision of the third chief division, and your other point will reach to 2,880, the product required.

### *Examples for Practice.*

1st. Multiply 20 by 3.
2d. Let it be required to multiply 16 by 3.
3d. Multiply 14 by 9.

### *Case 2d.*

To divide one number by another.

### *Rule.*

Extend your compasses from the divisor to 1, and that distance will reach from the dividend to the quotient.

### *Examples.*

1st. Divide 14 by 2.

Place one foot of your compasses at 2, and the other at 1; then that distance will reach from 14 to 7, the quotient.

2d. Suppose it be required to divide 140 by 10.

Extend your compasses from 10 to 1, and that will reach from 140 to 14, the quotient sought.

### *Examples for Practice.*

1st. Divide 56 by 4.

2d. Suppose it be required to divide 120 by 6.

3d. Let it be required to divide 200 by 4.

### *Case 3d.*

To perform the rule of three on the line of numbers.

### *Rule.*

The reader is supposed to be acquainted with the method of stating, and to perform the operations by figures; he will have nothing more to do than to extend his compasses from the first term to the third, and that distance will reach from the second term to the fourth.

### *Example.*

Required the value of 8 yards of cambric, when 3 cost 7 shillings. The proportion is this: 3 : 7 : : 8 to the value of 8 yards. Therefore extend your compasses from 3 to 8, and that extent will reach from 7 to $18\frac{2}{3}$ = 18s. 8d.

### *Examples for Practice.*

1st. What is the value of 9 coombs of corn, when 5 cost 6 pounds?

2d. Required the worth of 86 yards of silk, if 100 cost 78 pounds.

3d. Shew the price of 1 ell of muslin, when 34 ells of the same cost 11 pounds.

## DESCRIPTION OF THE LINES OF SINES, TANGENTS, &c.

The construction and use of these lines have been treated of before; being the same as the sectorial, it remains therefore only to show the manner in which they are drawn. First, then, the line of sines marked S. begins at the left hand, and is reckoned towards the right, numbered at each degree from 1 to 10 in the middle, and from 10, at every tenth degree, to 90, at the end. The subdivisions from 1 to 10 degrees are generally ten minutes, from 10 to 20 the subdivisions are each half a degree, from 20 to 30 the subdivisions are each a degree, from 30 to 70 each two degrees, from 70 to 80 each five degrees, and from 80 it is numbered to 90.

The next is the line of tangents marked T. beginning at the left hand at 1, and numbered at every degree up to 10 in the middle; from 10 in the middle it is numbered at every tenth degree up to 45. The subdivisions are the same as those on the line of sines; except that from 30 to 40, each subdivision is a degree.

On the other face of the instrument is a scale of inches beginning at the right hand, and numbered at every inch towards the left hand; every inch is divided into 10 parts. On the edge of the sector is also a scale of equal parts, divided into 10 parts, beginning at the right hand, and reckoned towards the left, each division $1\frac{1}{3}$ inch, and is also divided into 10 more equal parts; the numbers are 10, 20, 30, &c. to 100.

## MISCELLANEOUS QUESTIONS.

1st. Take 38 from the scale of equal parts.

2d. It is required to take 390 from the scale.

3d. Make an angle that shall contain 48 degrees.

4th. It is required to make a triangle, whose three angles are 60, 82, and 38.

5th. Let it be required to take 64 degrees from the line of sines.

6th. It is required to make a triangle, one angle of which is 38, and the other two equal.

7th. Let it be required to take 264 from the diagonal scale.

8th. Take 2135 from the diagonal scale.

9th. Take 3860 from the same scale.

10th. Multiply 38 by 5 on the line of numbers.

11th. Divide 190 by 5 on the same line.

12th. Divide a given line into 11 equal parts.

13th. Given the hypothenuse of a right-angled triangle and one of the acute angles, to find the base and perpendicular.

14th. A ship sails S. E. until her difference of latitude becomes 18 miles; required the departure and distance.

15th. Given the length of the radius of a circle 12 and the arc, 28 degrees; required the length of the sine, tangent, and secant.

16th. Given three sides of a triangle to determine the angles.

17th. Let it be required to take 62 degrees from the line of tangents.

18th. Take 47 degrees from the lines of chords, sines, tangents, and secants.

19th. Repeat the rule for performing the rule of three on the line of numbers.

20th. Given the perpendicular 39, and one of the acute angles 36 degrees; to find the base, and perpendicular.

21st. Repeat the rule for multiplication and division, by the line of numbers.

22d. Required the manner of constructing lines of sines, tangents, and secants.

23d. Divide a given line in the proportion of 2, 4, 6, and 7.

24th. Suppose a given line is divided into 6 parts in a given proportion, and it is required to divide another given line in the same proportion.

25th. Divide the given line into 16 equal parts.

26th. Let it be required to divide a given line in the proportion of 2, 4, 5, and 7.

27th. It is required to take 8, 4, 5, from the diagonal scale.

28th. Take 1, 8, 9, 6, from the small diagonal scale.

29th. Make a triangle whose angles are 72, 69, and 39, and one side opposite the first angle 298.

30th. Required the height of a tower, when an observer standing 84 yards from its bottom, found the angle formed by the top of the tower and the horizon to be 48 degrees.

31st. Multiply 95 by 6.

32d. It is required to make an angle which shall contain 62 degrees.

33d. Repeat the rule for determining the number of degrees in any given angle.

34th. Repeat the rule for making an angle, which shall contain a given number of degrees.

35th. Divide 85 by 5 on the line of numbers.

36th. Given the base, and one of the acute angles of a right-angled triangle, to find the hypothenuse and perpendicular.

37th. What the value of 28 yards of broad cloth, when 12 cost four pounds?

38th. Required the base and perpendicular of a right-angled triangle, whose hypothenuse is 21·7 and the acute angles 32, and 58 degrees.

39th. It is required to make a parallelogram, whose angles are given.

40th. Find how many degrees are contained in the angles of a given rhombus.

FINIS.

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